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INFLUENCE OF VIBRATIONS OF NON-NEWTONIAN FLUID FLOW

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The influence of vibrations on the exit flow of non-Newtonian power-law fluids from a vessel through capillaries of various diameters is investigated. It is found that the superposition of vibrations can either decrease or increase the time to empty the vessel.

The laminar pulsating motion of non-Newtonian power-law fluids in tubes has been investigated earlier [1,2]. It was found that the superposition of pressure pulsations on a steady non-Newtonian fluid flow in tubes can have the effect of increasing the average flow rate over one oscillatory period for pseudoplastic fluids and decreasing it for dilatant fluids.

We have conducted experimental investigations on an apparatus consisting of a cylindrical glass vessel, at the bottom of which was fused in a glass tube with a length of 15 cm and a diameter varying from 1 to 5 mm. The apparatus was thermostatically regulated and placed on a shake table. Vibrations were generated electromechanically in the frequency range from 0 to 70 Hz with an amplitude of 1 mm. The time for a specified quantity of fluid to flow out through capillaries of various diameters at a uniform temperature of 25°C was determined.

The experimental liquids were solutions of polyacrylamide (PAA) of various concentrations with the addition of 1% surfactant of the type DS-RAS, as well as petroleum. The rheological curves for flow of the investigated liquids show that they can be regarded as pseudoplastic non-Newtonian fluids.

The experimental results are shown in Figs. 1a-c, which give the exit time of a specified quantity of liquid from the vessel through capillaries of various diameters as a function of the vibration frequency. The concentrations of the PAA solution and the capillary diameters are indicated in the figure caption.

The results indicate that the superposition of vibrations on the exit-flow of the investigated liquids is effective at low vibration frequencies (around 10 Hz) and for flow through small-diameter capillaries. In the flow of PAA solutions with concentrations of 1% or higher through capillaries with diameters of 3 mm or more, the vibrations have scarcely any influence on the flow process. The effect of the vibrations vanishes for the investigated petroelum when the capillary diameter is increased above 4 mm.

We now analyze theoretically the flow of non-Newtonian fluids obeying a rheological power law from a vessel through vertical tube fused into its bottom.

The equation of motion of the fluid in the tube has the form

$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial z} - \frac{1}{r}\{r\tau_{rz}\} + \rho g, \quad (1)$$

$$\tau_{rz} = k\left(-\frac{\partial v}{\partial r}\right)\left|\frac{\partial v}{\partial r}\right|^{n-1}. \quad (2)$$

At the initial time t = 0, the flow velocity of the fluid is equal to zero. For t > 0,

$$-\frac{\partial p}{\partial z} = \frac{\rho g h(t)}{l}, \quad (3)$$

where h(t) is the level of the fluid in the vessel and l is the length of the tube. Substituting relations (2) and (3) into Eq. (1), we obtain

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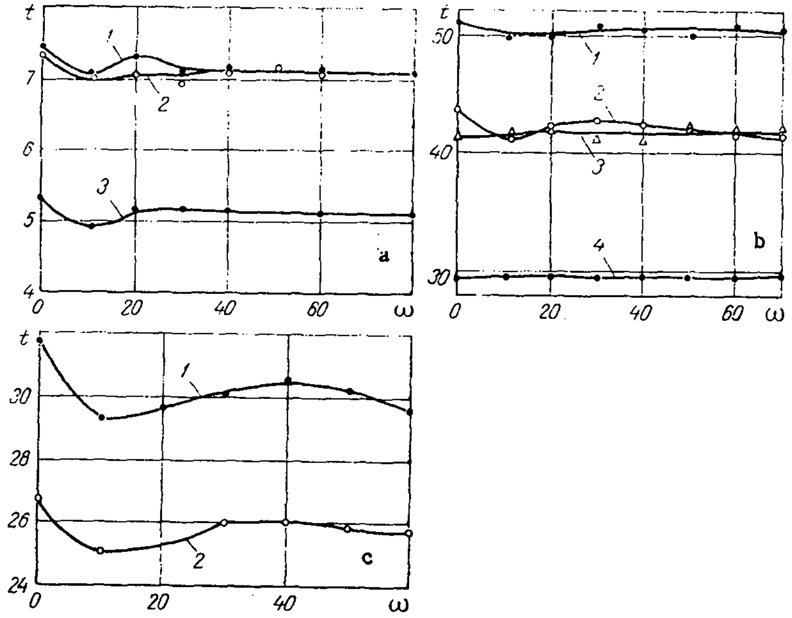

Fig. 1. Exit time t, min (a) or sec (b, c) versus vibration frequency, Hz. a: 1) PAA concentration 1.5%, capillary diameter 2 mm; 2) petroleum, 4 mm; 3) 0.5% PAA, 1 mm. b: 1) 2% PAA, 4 mm; 2) 0.5% PAA, 2 mm; 3) 1% PAA, 3 mm; 4) 2% PAA, 5 mm. c: 1) 1% PAA, 1 mm; 2) petroleum, 3 mm.

$$\rho \frac{\partial v}{\partial t} = \varphi(t) + \frac{1}{r}\frac{\partial}{\partial r}\left\{rk\left(\frac{\partial v}{\partial r}\right)\left|\frac{\partial v}{\partial r}\right|^{n-1}\right\},$$
$$\varphi(t) = \rho g\left[\frac{h(t)}{l} + 1\right]. \quad (4)$$

To determine the unknown fluid level h(t), we use the equation

$$S\frac{dh}{dt} = q(t). \quad (5)$$

The volumetric flow rate q(t) is given by the expression

$$q(t) = 2\pi\int_0^a vr\,dr. \quad (6)$$

The boundary conditions for Eq. (4) have the form

$$r = a, \quad v = A\cos\omega t,$$
$$r = 0, \quad \partial v/\partial r = 0. \quad (7)$$

We consider the case in which the viscous forces are much greater than the inertial forces; then the inertial term can be neglected in (4), which acquires the form

$$\varphi(t) + \frac{1}{r}\frac{\partial}{\partial r}\left\{rk\left(\frac{\partial v}{\partial r}\right)\left|\frac{\partial v}{\partial r}\right|^{n-1}\right\} = 0. \quad (8)$$

We also assume that dv/dr does not change sign, and, integrating Eq. (8) subject to the boundary conditions (7), we obtain

$$v = A\cos\omega t + \left[\frac{\varphi(t)}{2k}\right]^{1/n}\frac{n}{n+1}\left[a^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right].$$

The flow rate is given by the relation

$$q(t) = \pi a^2\left\{A\cos\omega t + \left[\frac{\varphi(t)}{2k}\right]^{1/n}\frac{n}{(3n+1)}a^{1+\frac{1}{n}}\right\}. \quad (9)$$

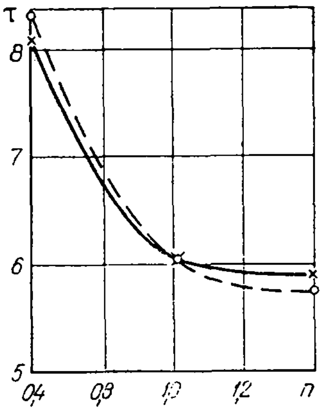

Fig. 2. Dependence of the dimensionless time to empty vessel on the rheological parameter n for $A_2 = 0.1$ and different values of the parameter A_1. Solid curve $A_1 = 0.1$, dashed curve $A_1 = 0$.

For the determination of the fluid level h(t) in the vessel, we obtain from (5) and (9)

$$-S\frac{dh}{dt} = \pi a^2\left\{A\cos\omega t + \left[\frac{\varphi(t)}{2k}\right]^{1/n}\frac{n}{(3n+1)}\,a^{1+\frac{1}{n}}\right\}. \tag{10}$$

We introduce the dimensionless variables

$$\xi = h/l \text{ and } \tau = \omega t/2\pi. \tag{11}$$

Knowing that the average velocity v_{av} for steady flow of a non-Newtonian power-law fluid in a tube is related to the constant pressure gradient φ_0 by the equation

$$v_{av} = \left(\frac{\varphi_0}{2k}\right)^{1/n}\frac{na^{1+\frac{1}{n}}}{(3n+1)}, \quad \varphi_0 = \frac{\rho g H}{l} + \rho g,$$

we obtain from (10) and (11)

$$-\frac{d\xi}{d\tau} = A_1\cos 2\pi\tau + \frac{A_2}{\left(1+\frac{H}{l}\right)^{1/n}}(\xi+1)^{1/n}. \tag{12}$$

Here

$$A_1 = \frac{\pi a^2}{S}\,\frac{2\pi}{\omega}\,\frac{A}{l}, \quad A_2 = \frac{\pi a^2}{S}\,\frac{2\pi}{\omega}\,\frac{v_{av}}{l}.$$

We solved Eq. (12) numerically on a computer. The results of the calculations (Fig. 2) show that for specified values of the parameters of the problem ($A_2 = 0.1$, $0.5 \geq \xi \geq 0$) the time to empty the vessel in the presence of vibrations (solid curve, $A_1 = 0.1$) decreases for $n < 1$ (pseudoplastic fluids) and increases for $n > 1$ (dilatant fluids). The dashed curve corresponds to $A_1 = 0$ and $A_2 = 0.1$, i.e., vibrations are absent. We have thus determined the fact that the superposition of vibrations on the exit-flow process in the case where the inertial forces can be neglected in comparison with the viscous forces (as is true for low vibration frequencies) can result in a decrease of the vessel emptying time for pseudoplastic fluids and an increase in that time for dilatant fluids. These theoretical results are consistent with the experimental.

NOTATION

ρ, density; v, velocity; t, time; p, pressure; z, coordinate measured along tube axis; r, coordinated measured along tube radius; τ_{rz}, tangential stress; g, free-fall acceleration; n, k, rheological parameters of fluid; S, cross-sectional area of vessel; a, tube radius; ω, frequency; A, amplitude; ξ, dimensionless fluid level in vessel; τ, dimensionless time; H, initial level of fluid in vessel.

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THEORY OF A PERIODIC LAMINAR BOUNDARY LAYER

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A method is proposed for the analysis of a periodic laminar boundary layer, refining the conventional methods of Lin, Rayleigh, and Hill and Stenning and providing a basis for the unification of those methods.

Derivation of the Fundamental System of Equations. The equations for a periodic laminar boundary layer have the form

$$\left.\begin{aligned} \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2},\\ \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\end{aligned}\right\} \quad (1)$$

$$u=0;\ v=0 \text{ at } y=0;\ u\to U(x,\ t) \text{ as } y\to\infty;$$
$$u=f(y,\ t) \text{ at } x=x_f. \quad (2)$$

The velocity at the outer boundary of the boundary layer is given by the expression

$$U(x,\ t)=U_0(x)+W(x)\cos(\omega t).$$

The absence of a temporal boundary condition in the case of steady-state periodic motion renders it impossible, in principle, to solve the problem directly. This fact makes it necessary to adopt a specific representation of the time dependence of the functions u and v.

We investigate the expansions of these functions in Fourier series, written in complex form:

$$\left.\begin{aligned} u=u_0(x,\ y)+\mathrm{Re}\cdot\sum_{s=1}^{\infty}u_s(x,\ y)\exp(si\omega t),\\ v=v_0(x,\ y)+\mathrm{Re}\cdot\sum_{s=1}^{\infty}v_s(x,\ y)\exp(si\omega t).\end{aligned}\right\} \quad (3)$$

Here u_0 and v_0 are unknown real functions, and u_s and v_s are unknown complex functions. The functions u and v can be represented by Fourier series, since they satisfy the sufficient conditions for expansion (periodicity with respect to time and differentiability at any point of the domain of definition); see the system (1) and the boundary conditions (2). Assuming sufficiently rapid convergence of the series (3), hereinafter we use segments thereof containing only two harmonics. We substitute these segments into the system (1), writing the velocity at the outer boundary of the boundary layer in the form $U(x,\ t) = U_0(x) + \mathrm{Re}[W(x)\cdot\exp(i\omega t)]$. To take the operator Re for extraction of the real part outside the multiplication sign, we invoke the formula

$$\mathrm{Re}z_1\mathrm{Re}z_2=\frac{1}{2}\mathrm{Re}(z_1z_2+z_1\bar{z}_2).$$

The overbar is used everywhere to denote the complex conjugate, and z_1 and z_2 denote arbitrary complex numbers. After the appropriate calculations, the first equation of the system (1) can be written

$$\sum_{p=0}^{4}\mathrm{Re}[N_p(u_0,\ u_1,\ u_2,\ v_0,\ v_1,\ v_2)\exp(pi\omega t)]=0, \quad (4)$$

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